

Recall  $b^0 = 1$  and  $b^1 = b$   
on a number line  $\rightarrow$  means for subtraction

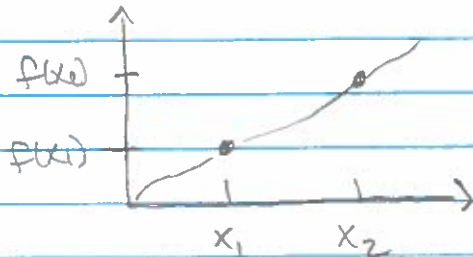
## Increasing & decreasing fcts:

defn: a function is increasing...

whenever

$$x_1 < x_2 \text{ then } f(x_1) < f(x_2)$$

what does that look like:



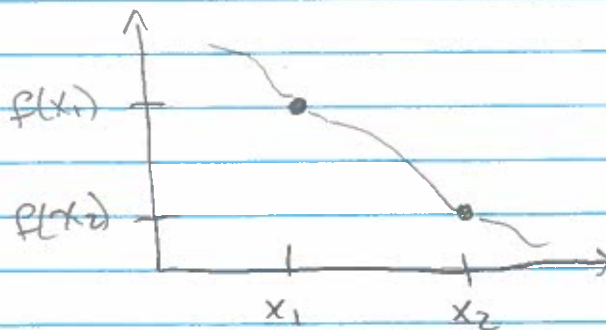
Remember to use colors!

defn: a function is decreasing

whenever

$$x_1 < x_2 \text{ then } f(x_1) > f(x_2)$$

what does that look like:



what does  
(Avg) Rate of change

have to say about increasing & decreasing

Increasing:

$x_1 < x_2$  means  $x_2 - x_1 > 0$  {positive}

$$\frac{\Delta y}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1} > 0$$

{positive}

increasing:

$f(x_1) < f(x_2)$

means

$f(x_2) - f(x_1) > 0$

{positive}

+

Decreasing:

$$\frac{\Delta y}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1} < 0$$

{negative}

decreasing:

$f(x_1) > f(x_2)$

means

$f(x_2) - f(x_1) < 0$

{negative}

-

\* Remember  
this!

Very important!

percent change:  
(relative)

Book always means this!

\* {w/ step size 1} \*  $\rightarrow$  i.e.  $\Delta x = 1$

$$\frac{\Delta y}{y_0}$$

e.g. if we increase cost depends on original cost if it's a "big" change

(a) a gallon of gas costing \$2.25

(b) a cell phone costing \$180

• For both calculate the percent change \* {step size 1} \* for an increase of \$2

$$\frac{2.25 - 2.00}{2.25} = \frac{.25}{2.25} = \frac{1}{9} \quad \left| \quad \frac{180 - 2}{180} = \frac{178}{180}$$

# Applications to economics:

defn

The cost function

gives total cost of producing a quantity "q" of some good

notation:  $c(q)$

\* first we will assume the cost of a "unit" is constant \*

$$\text{Total cost} = \text{fixed costs} + (\text{cost of a unit}) \cdot q$$

a cost that doesn't change

how much it costs to make 1 "unit" → employee and raw

like:

- rent
- lights etc...

e.g. consider a company that makes TVs. The Factory and Machinery needed to begin producing TVs and occurs even when no TVs are made, has a cost of \$24,000. The cost of labor and raw materials is \$10 per TV.

- what are the "fixed costs", "cost of a unit"
- write the equation for  $c(q)$
- graph the equation



defn marginal cost is the <sup>(av.)</sup> rate of change  
\* { speed cost increases/decreases }

• what is the marginal cost of the preceding example?

A cost function is completely defined by total and fixed costs

defn Revenue function Total revenue received for selling quantity "q" of a good notation:  $R(q)$

Revenue = price  $\cdot$  quantity

eg Consider the TV example from before

• Say you sell a TV for \$200 dollars when do you "break even"  $R(q) = 200q$

\* draw the graphs \*

\* ask the question: "what does "break even"

Finally do some algebra to solve for intersection

$$200q = 2400 + 10q \quad * \text{ solve for } q *$$

\* remember to use color \*

profit fun.  
 $P(q) = R(q) - C(q)$

## Supply & demand:

### Supply curve:

input  
↓  
Supply is a function of price ( $P$ )  
and the output is how much ( $Q$ ) quantity the producer  
willing to make @ that price

### demand curve:

input  
↓  
demand is a function of price ( $P$ )  
and the output is how much ( $Q$ ) quantity "the market  
is willing to purchase @ that price


\* defn

we say the Market is @ equilibrium  
when

$$\text{Supply} = \text{demand}$$

and call the  $(P_0, Q_0)$  where these meet

the equilibrium price & quantity

\* draw a pic: 

eg: find the equilibrium price

$$\text{if } \text{Supply} = 3P - 50$$

$$\text{demand} = 100 - 2P$$

$$3P - 50 = 100 - 2P$$

Start here!

Recall

exponent

Rules:

$$a^m \cdot a^n = a^{m+n}$$

$$(a^m)^n = a^{m \cdot n}$$

$$\frac{a^m}{a^n} = a^{m-n}$$

$$a^0 = 1$$

$$\frac{1}{a^n} = a^{-n}$$

\* keep  
on the  
board

Use color!

always!



# Exponential functions:

example:

population of New York

	year	pop. (mill)
$t=0$	2003	12.853
$t=1$	2004	13.290
$t=2$	2005	13.747
$t=3$	2006	14.226
$t=4$	2007	14.721
$t=5$	2008	15.234
$t=6$	2009	15.757

let's look @ (avg.) rate of change

$$\frac{13.290 - 12.853}{1 - 0} = 0.437$$

draw a graph!

$$\frac{13.747 - 13.290}{2 - 1} = 0.457$$

So Not a line

Since Not constant (avg.) rate

Now look @

relative (percent) change [step size 1]

$$\frac{13.290 - 12.853}{12.853} = 0.034$$

$$\frac{13.747 - 13.290}{13.290} = 0.034$$

$$\frac{14.226 - 13.747}{13.747} = 0.034$$

} all equal

So Constant relative (percent) change [w/step size 1]

Last time w/ constant (avg) rate of change we got a line  
what do we get now? f



So we want a function

$P(t)$

initial population

we just saw:  $P(0) = 12,853$  and...

$$\frac{P(1) - P(0)}{P(0)} = 0.034$$

$$\Rightarrow P(1) - P(0) = P(0) \cdot (0.034)$$

$$\Rightarrow P(1) = P(0) \cdot (0.034) + P(0)$$

$$\Rightarrow P(1) = P(0) (1 + 0.034) = P(0) (1.034)$$

and  $\frac{P(2) - P(1)}{P(1)} = 0.034$

$$\Rightarrow P(2) - P(1) = P(1) (0.034)$$

$$\Rightarrow P(2) = P(1) (0.034) + P(1)$$

$$\Rightarrow P(2) = P(1) (1 + 0.034) = P(1) \cdot (1.034)$$

$$= P(0) (1.034) (1.034) = P(0) (1.034)^2$$

and  $\frac{P(3) - P(2)}{P(2)} = 0.034$

$$\Rightarrow P(3) - P(2) = P(2) (0.034)$$

$$\Rightarrow P(3) = P(2) (1.034) = P(0) (1.034)^2 (1.034)$$

$$= P(0) (1.034)^3$$

so  $D(t) = P(0) (1.034)^t = 12,853 (1.034)^t$

## The natural log

euler's # (pronounced oiler's #)  
 $e = 2.718\dots$

Q: So what's the opposite  
of  $e^x$ ?

like  $-3$  is the opposite of  $+3$

and  $\frac{1}{3}$  is the opposite of times 3

Ans:  $\ln$  {the natural log}

has the "opposite rules as exponents"

Rules of LN

$$\ln\left(\frac{A}{B}\right) = \ln(A) - \ln(B)$$

$$\ln(A \cdot B) = \ln(A) + \ln(B)$$

$$\ln(A^p) = p \ln(A)$$

$$\ln(e^x) = x$$

$$e^{\ln(x)} = x$$

$$\ln(1) = 0 \quad \text{since } e^0 = 1 \quad \text{and } \ln(e) = 1$$

\* there is  
no  $\ln$  of  
negative #'s  
or zero!

$$\text{So } e^{\ln(a)} = a$$

So we can write

$$a^t = (e^{\ln(a)})^t = e^{t \cdot \ln(a)}$$

So from before

$$P(t) = P_0 (1+r)^t = P_0 e^{t \cdot \ln(1+r)}$$

\* we call  $\ln(1+r)$  the continuous growth/decay

So in general:

growth/decay

$$P(t) = P_0 e^{k \cdot t}$$

initial  
population

where  $k = \ln(1+r)$

where

$r$  is the constant  
relative change



# Compound Interest

These things may sound foreign to you  
but banks use them every day

annual compounding interest  
and continuous compounding interest

this is the  
relative chr

this is the continuous rate

eg.) A bank offers 8% interest on a savings account, would you make more money if they compounded annually or continuously? [say you invested \$4000]

annually

$$P(t) = P_0 (1+r)^t = 4000(1.08)^t$$

$$\text{so } P(1) = 4000(1.08) \approx$$

Continuously

$$P(t) = P_0 e^{rt} = 4000 e^{0.08t}$$

so

$$P(1) = 4000 e^{0.08} \approx$$

## let's see decay

e.g.) when a patient is given a medication the drug enters the blood stream. The rate @ which the drug is metabolized and eliminated depends on the particular drug. For a certain antibiotic approximately 30% of the drug is eliminated every hour. A typical dose of this medicine is 200 m.g.

So  $Q(t)$  the quantity of the drug left in the blood stream after  $t$ -hours of injection

$$\text{is } Q(t) = Q_0 e^{k \cdot t}, \text{ where } Q_0 = 200 = Q(0)$$

and

$$r = \frac{Q(1) - Q(0)}{Q(0)} = -0.3$$

→ this is decay!  
→ why is it negative?

$$\text{So } k = \ln(1 - 0.3) = \dots$$

→ this continuous growth rate

$$Q(t) = Q_0 e^{\ln(1-0.3) \cdot t}$$

example During the 1980's, Costa Rica had the highest deforestation rates in the world at 2.9% per year (this is the rate that land covered by forests is shrinking) Assuming this rate continues what percent of the land in Costa Rica covered by forest in 1980 will still be covered in 2015?

[what are we looking for]  
 if  $f(t)$  is land covered by forests  
 we are looking for

$$\frac{f(35)}{f(0)}, \text{ well}$$

$$f(t) = f_0 (1 - 0.029)^t$$

So

$$\frac{f(35)}{f(0)} = (1 - 0.029)^{35}$$

example] Suppose  $f(t)$  is an exponential and  $f(20) = 88.2$  and  $f(23) = 91.4$

(a) find the continuous growth rate.

(b) when will it equal 100

Sol'n] (a)  $\frac{91.4}{88.2} = \frac{f(t) e^{k \cdot 23}}{f(t) e^{k \cdot 20}} = e^{3k}$

$$\frac{\ln\left(\frac{91.4}{88.2}\right)}{3} = k \approx \dots$$

(b) find  $f(0)$ ,  $88.2 = f(0) e^{k \cdot 20}$  so  $f(0) = \frac{88.2}{e^{k \cdot 20}}$

$\dots \dots \dots P(t) = P_0 e^{k \cdot t} \dots \dots \dots (100)$



## doubling time & half time

defn

doubling time in exponential growth is the time it takes to double

defn

half-life in exponential decay is the time it takes to get in half

example:

(a) if the continuous rate is 0.01  
what is the doubling time?

(b) if the continuous rate is -0.01  
what is the half life?

Sol'n (a)

$$\frac{2 \cdot P_0}{P_0} = \frac{P_0 e^{0.01 \cdot t}}{P_0} \Rightarrow 2 = e^{(0.01)t} \Rightarrow \frac{\ln(2)}{0.01} = t$$

$$(b) \frac{\frac{1}{2} P_0}{P_0} = \frac{P_0 e^{(-0.01)t}}{P_0} \Rightarrow \frac{1}{2} = e^{(-0.01)t} \Rightarrow \frac{\ln(\frac{1}{2})}{-0.01} =$$

$$\Rightarrow \frac{-\ln(2)}{0.01} = t \quad \underline{\underline{\text{what!!!}}}$$

They are  
the same

What!!!???

## Group work:

G.W. 1 The consumer price Index (CPI) for a given year is the amount of money in that year that has the same purchasing power as \$100 in 1983. At the start of 2009, the CPI was 211. Write a formula for the CPI as a function of years after 2009, assuming the CPI increases by 2.8% per year.

Soy bean production: (in millions)

<u>G.W. 2</u> year	2000	2001	2002	2003	2004
production	161.0	170.3	180.2	190.7	201.8

→ does this data suggest a line or an exponential function or neither? why?

→ can you write a function for this data? if so do!

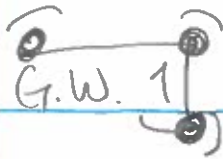
G.W. 3 An air freshener starts w/ 30 grams and evaporates 12% a day. write a formula for this decay. what is the air freshener's half life?

Chapter 1

Section 1.6

2.1 2.1

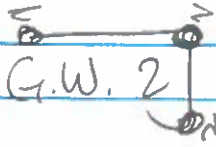
group work



G.W. 1

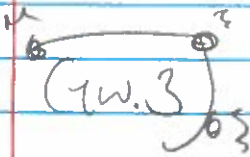
A cup of coffee contains 100 mg of caffeine, which leaves the body at a continuous rate of 17% per

what's the half life of caffeine in the body after drinking a cup of coffee?



G.W. 2

Prozac has a half life of about 3 days. what percentage is in your blood stream after 2 days?



G.W. 3

In 2012 the world's population was 7 and is projected to reach 8 billion by the year 2025. what annual growth rate is projected? what continuous rate is that?



## New functions from old:

\* A look to the future:

$$\text{Cost} = \text{fixed cost} + \text{variable cost}$$

$$\# + f(q)$$

↑ some function of quantity

So take our first example:

$$C(q) = b + mq \quad \{\text{baby example} / \text{base case}\}$$

then

$$C\left(\frac{1}{m}f(q)\right) = b + f(q) \quad \text{any other example}$$

\* Some can make any cost function "from" the baby example

Composition: \* don't forget colors \*

$$f(x) = x^2 + 7x + 2$$

$$f(2) = (2)^2 + 7(2) + 2$$

$$f(x+h) = (x+h)^2 + 7(x+h) + 2$$

\* what g(x)

$$f(g) = (g)^2 + 7(g) + 2$$

$$f(g(x)) = (g(x))^2 + 7(g(x)) + 2$$

